

Deformation and relaxation of viscous thin films under bouncing drops

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Thin, viscous liquid films subjected to impact events can deform. Here we investigate free surface oil film deformations that arise due to the air pressure buildup under the impacting and rebounding water drops. Using Digital Holographic Microscopy, we measure the 3D surface topography of the deformed film immediately after the drop rebound, with a resolution down to 20 *nm*. We first discuss how the film is initially deformed during impact, as a function of film thickness, film viscosity, and drop impact speed. Subsequently, we describe the slow relaxation process of the deformed film after the rebound. Scaling laws for the broadening of the width and the decay of the amplitude of the perturbations are obtained experimentally and found to be in excellent agreement with the results from a lubrication analysis. We finally arrive at a detailed spatio-temporal description of the oil film deformations that arise during the impact and rebounding of water drops.

1. Introduction

Drops impacting a liquid layer frequently occurs in nature as well as in many industrial and technological applications. Common examples are raindrops hitting the surface of a pond, spray coating on a wet substrate or inkjet printing on a primer layer. The collisions can generate complex scenarios such as floating, bouncing, splashing or jetting, which have been extensively studied (Worthington 1908; Rein 1993; Weiss & Yarin 1999; Thoroddsen *et al.* 2008; Ajaev & Kabov 2021). Impact velocity, impact angle, droplet size, liquid layer thickness, and the material properties of the liquids are the parameters which determine the impact dynamics. Among the many different impact scenarios, a particularly intriguing phenomenon is reported to occur at sufficiently low impact velocities: floating or bouncing drops which never directly contact the underlying liquid. The earliest reported observation of a drop floating over a liquid surface was made by Reynolds (1881) who noticed that under certain circumstances, the drops spraying from the bow of a boat or droplets from a shower of raindrops float on liquid surfaces for some seconds before they disappear. Later, Rayleigh (1882) reported bouncing of drops when collision of two distinct streams of liquids resulted in, under certain circumstances, drops bouncing off each other without merging. The

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reason for the presence of repulsion forces on impacting droplets even without direct contact with the (liquid) substrate is a lubrication pressure build-up in the draining thin air layer between droplet and substrate which was first detailed in the theoretical work by [Smith *et al.* \(2003\)](#). The importance of such thin air layers sparks interest in numerous recent investigations of eg., skating drops ([Mandre *et al.* 2009](#); [Hicks & Purvis 2010, 2011](#); [Kolinski *et al.* 2012, 2014](#)), entrapment of bubbles ([Thoroddsen & Takehara 2003](#); [Thoroddsen *et al.* 2005](#); [Tran *et al.* 2013](#); [Hendrix *et al.* 2016](#)), dimple formation under a falling drop ([van der Veen *et al.* 2012](#); [Duchemin & Josserand 2012](#); [Li & Thoroddsen 2015](#)), and suppressing of splash ([Xu *et al.* 2005](#)). Floating/bouncing drops can be observed not only on liquid surfaces but also on dry surfaces. Such scenarios include drops bouncing on a dry surface ([Kolinski *et al.* 2012, 2014](#)), drops floating on a very hot surface (Leidenfrost effect) ([Chandra & Avedisian 1991](#); [Quééré 2013](#)), drops bouncing on a pool of liquid ([Rodriguez & Mesler 1985](#); [Klyuzhin *et al.* 2010](#)), drops floating/bouncing on a vibrating pool of liquid ([Couder *et al.* 2005a,b](#)), and drops floating on a very cold pool of liquid (inverse Leidenfrost effect) ([Gauthier *et al.* 2019a,b](#)).

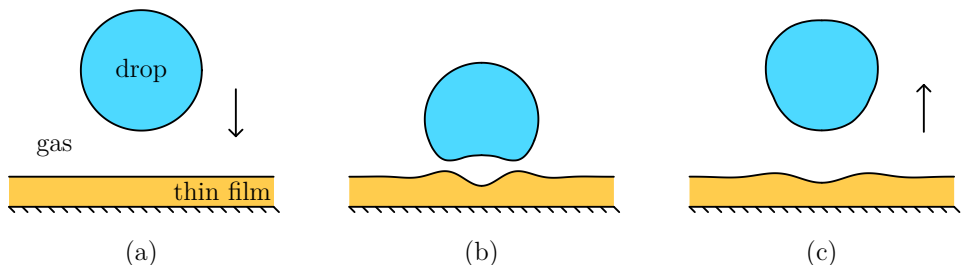


Figure 1: Schematic diagram (not to scale) of a drop bouncing on a thin film in a surrounding gas environment. Three stages in the bounce process are shown: (a) Prior to impact, (b) during the bounce, where both the droplet and the oil film deform, (c) after the bounce, where the oil deformations slowly relax.

In the present study, we investigate a drop bouncing on a thin liquid film. A schematic diagram is shown in figure 1 highlighting three important stages of a drop bouncing scenario. (a) Initial stage (cf. figure 1a) - A drop falls towards a flat film surface in a surrounding gas medium. (b) Deformation stage (cf. figure 1b) - The drop's center of mass velocity changes direction due to the lubrication force provided by the narrow gas layer separating the two liquids which exceeds the droplet's weight. Large spatial variations of the gas pressure cause large drop and significant thin film deformations in this stage. (c) Relaxation stage (cf. figure 1c) - The drop is far from the film surface after the bounce. The gas pressure separating the two liquids is again reduced to ambient pressure and the thin film deformations gradually decay via an intricate relaxation process.

Important parameters for the study of drops bouncing on thin liquid layers are the initial depth or height h_f of the liquid layer above an underlying solid substrate and the drop radius R_w . Experiments by [Pan & Law \(2007\)](#) reveal drop bouncing to be favoured on deep pools which have $h_f > R_w$, as compared to on thick liquid layers which have $h_f \approx R_w$ and to on thin films which have $h_f < R_w$. It was argued that the solid substrate (wall) restricts the penetration of the falling drop in the thin films, thereby

suppressing bouncing. For thin films, the bouncing phenomenon is only observed for drops having moderately low kinetic energy as compared to their surface energy, i.e., $We = \rho_w R_w v_w^2 \gamma_w^{-1} \lesssim 10$, where ρ_w is the density of the drop, v_w the drop impact speed and γ_w the surface tension of the drop. At sufficiently high impact velocities, the drop contacts the underlying liquid due to the Van der Waals attraction force between the two liquids. This effect becomes important when the liquid-liquid separation is smaller than around 100 nm (Charles & Mason 1960). The critical Weber number which marks the transition from drop bouncing to merging has been studied by Tang *et al.* (2018), using liquids of different viscosities. They found that the critical Weber number, below which the drop bounces, increases as the liquid viscosity (drop and the thin film) and the thin film thickness are increased. This finding indicates that larger viscosity liquids promote drop bouncing. Similar observations are made in the work of Langley & Thoroddsen (2019), where delayed coalescence is observed for drops and thin films with large viscosities.

Gilet & Bush (2012) and Hao *et al.* (2015) found drop bouncing on a thin film to be similar to bouncing on a super-hydrophobic substrate. One such similarity was the apparent contact time of the drop which agreed well with the Hertz contact time (Richard *et al.* 2002). However, the droplet-film collision resembled an almost elastic collision between the two liquids with the coefficient of restitution close to unity. Pack *et al.* (2017); Lo *et al.* (2017); Tang *et al.* (2019) used interferometry measurements to obtain the time-resolved evolution of nanometric profiles of the air gap between impacting drops and thin viscous layers. They found a bell shaped annular air profile with maximum thickness at the center and minimum thickness at a radially outwards location which varied with time. Small variations in air profiles were observed when the impacting drop was slightly oblique relative to the underlying film surface and when the film thickness was increased from thin film to a deep pool limit. Significant asymmetries were also observed in the evolution of air profiles when comparing the drop spreading stage to the receding stage for a typical bounce process. Lo *et al.* (2017) successfully measured both the drop and the thin film deformations during the approach process. The thin film and the air film deformations were measured using the high-speed confocal profilometry technique and the dual-wavelength interferometry technique respectively. The drop deformation was inferred from the thin film and the air film deformations at around the same time instance by performing two separate experiments under identical impact conditions. The limitation in their measurement is that the thin film deformations had a $1.8 \mu\text{m}$ vertical resolution and that they could only be obtained for a few time instances before the rupture of the air film.

Previous experimental and numerical studies of drop bouncing on thin films mainly focused either on the macroscopic drop bouncing behaviour or on the evolution of the nanometric gas thickness between the two liquids without providing a distinction between drop and thin film deformations. All experimental studies except Lo *et al.* (2017) ignore the thin film deformations owing to the small film thickness and large film viscosity used in the experiments (Pack *et al.* 2017; Gilet & Bush 2012). They rather assume that the thin liquid film mimics a perfectly smooth solid surface. The numerical studies of thin film deformations prove challenging because of the large difference in involved length scales (millimetric to nanometric deformations) when computing the lubrication gas flow and the thin film flow simultaneously to drop deformations (Josserand & Zaleski 2003). Although viscous thin film deformations are typically small, they cannot be neglected since they play a crucial role in modulating the gas layer thickness, thereby affecting

the drop bouncing process and possibly the coalescence of the drop with the thin film at higher impact velocities.

Finally, the thin film deformations can also give insight into the size and the velocity of the impacting drop, much alike how impact craters are used to determine the size and velocity of the impacted body. Understanding the size and dynamics of the thin film deformations will allow for a design of liquid infused surface (Quéré 2008) which can reduce lubricant depletion through shearing, cloaking and in the wetting ridge (Smith *et al.* 2013; Schellenberger *et al.* 2015; Kreder *et al.* 2018).

The objective of the paper is to measure oil film deformation that arise due to impacting and rebounding water drops in an ambient air environment. The impacting water drops have $We \sim 1$ when inertial and capillary forces roughly balance so that the bouncing actually occurs. Thanks to Digital Holographic Microscopy we achieve the unprecedented precision down to 20 nm in the vertical resolution at 0.5 kHz recording speed. To our knowledge, the sub-micrometer thin film deformations reported in this experimental work are the first deformation measurements that explicitly document the effect of the air pressure buildup under impacting and rebounding drops.

The structure of the paper is as follows: First, in section 2, the experimental setup and the control parameters are described and some typical orders of magnitude of the relevant non-dimensional numbers are given. The subsequently presented results are twofold: In section 3, we discuss the film surface deformations immediately after the bouncing event. We quantify how the surface deformations depend on the film thickness, film viscosity and drop impact speed. This part of the paper describes the deformations of the thin film after the end of the deformation stage (cf. figure 1b). The second part of our study is presented in section 4 and focusses on the relaxation stage described above (cf. figure 1c). We first illustrate a typical relaxation process of film deformations which occur after the drop bouncing. Starting from experimentally obtained deformations as initial conditions, we then compare the evolution of the experimental profiles in the relaxation process to a numerical calculation using lubrication theory. Next, we use a general theoretical result of Benzaquen *et al.* (2015) for the relaxation of thin film deformations. The thus obtained scaling laws for the width broadening $\lambda(t)$ and amplitude decay $\delta(t)$ during the relaxation process are compared to our experiments over a large range of parameters. Finally, the paper closes with a Discussion in section 5.

2. Experimental details

A schematic drawing of the experimental setup is shown in figure 2. Using a syringe pump, Milli-Q water is slowly dispensed out of a needle tip as soon as the droplet's weight overcomes the surface tension force. The detached water droplet of radius $R_w = 1.08$ mm is made to fall on a thin and viscous silicone oil film. The 3D surface topography of the deformed oil film is measured using a digital holography technique, as described below, complemented by simultaneous side view visualisations of the drop dynamics. The silicone oil films are prepared by the method of spin coating on cleaned glass slides. The film thickness is measured by the use of a white light spectrometer. Table 1 gives the density, dynamic viscosity, and surface tension of different liquids used in the present study. The density and dynamic viscosity of air at standard temperature and pressure are $\rho_a \approx 1.2$ kg/m³ and $\eta_a \approx 0.002$ mPa.s, respectively. The film deformations are measured by varying three important control parameters, namely the oil film thickness

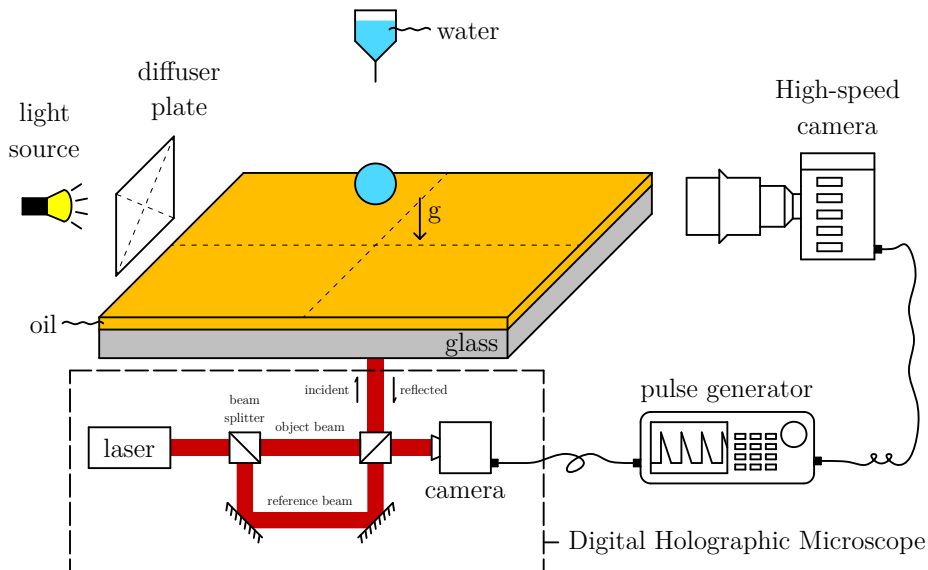


Figure 2: Schematic diagram (not to scale) of the experimental setup. Bottom view: Holographic setup, positioned underneath the glass substrate. It is used to measure free surface oil film deformations. Side view: The dynamics of the impacting water drop is characterised with a high speed camera.

h_f which is either about $5 \mu\text{m}$, $10 \mu\text{m}$, or $15 \mu\text{m}$, the oil film viscosity η_f which is either $52 \text{ mPa}\cdot\text{s}$, $98 \text{ mPa}\cdot\text{s}$, or $186 \text{ mPa}\cdot\text{s}$, and the impacting water droplet speed v_w , for which we choose 0.16 m/s or 0.37 m/s .

Liquids	Density ρ [kg/m^3]	Dynamic viscosity η [$\text{mPa}\cdot\text{s}$]	Surface tension γ [mN/m]
Water (w)	995	1	72
Silicone oils (f)	959	52	20
	963	186	19

Table 1: Properties of liquids used in the experiments. Subscripts w and f represent water and oil-film respectively. γ is the liquid-air surface tension.

For the measurement of film deformations, a holographic technique is used (Gabor 1949; Schnars *et al.* 2016). The Digital Holographic Microscopy (DHM[®]-R1000 by Lyncée Tec) device provides real-time measurements of (at least) 20 nm vertical resolution within the $200 \mu\text{m}$ measuring window (cf. Appendix A). A 2.5x objective is used along with the DHM setup, which provides a roughly $4.90 \mu\text{m}$ lateral resolution and allows for measurements of a maximum deformation slope up to 2° . A pulse generator connects and synchronizes the recordings of the side view camera and the DHM camera at a temporal resolution of around 0.5 kHz .

Deformation	Relaxation
$We = \rho_w R_w v_w^2 \gamma_w^{-1} \sim 1$	$Re_f = \rho_f h_f \gamma_f \eta_f^{-2} \sim 10^{-2}$
$Re_w = \rho_w R_w v_w \eta_w^{-1} \sim 10^2$	$\eta^* = \eta_f \eta_w^{-1} \sim 10^2$
$Re_a = \rho_a R_w^{1/2} h_a^{1/2} v_w \eta_a^{-1} \sim 10^{-1}$	$h^* = h_f R_w^{-1} \sim 10^{-2}$

Table 2: Relevant dimensionless numbers and their orders of magnitude for both the deformation and the relaxation stage. Subscripts w , a , and f represent water, air, and oil-film, respectively.

Given the experimental parameters stated in this section, the orders of magnitude of the relevant dimensionless numbers are summarized in table 2. The dimensionless numbers are useful to identify some qualitative flow features for the deformation stage (cf. figure 1b) and the relaxation stage (cf. figure 1c) pertaining to drop bouncing. For the deformation stage, the low value of the Weber number $We \sim 1$ causes drops to bounce on viscous thin films (Hao *et al.* 2015; Lo *et al.* 2017; Pack *et al.* 2017; Tang *et al.* 2019). The Reynolds number $Re_w \sim 10^2 \gg 1$ of the flow in the drop and the absence of no-slip boundary conditions allow for the applicability of potential flow theory inside the bouncing drop independent of the substrate underneath (Moláček & Bush 2012; Hendrix *et al.* 2016). The low Reynolds number $Re_a = \rho_a L_a v_a \eta_a^{-1} \sim 10^{-1}$ of the air flow indicates a viscous squeeze flow in thin air gaps during impact. Here $L_a \sim R_w^{1/2} h_a^{1/2}$ is the length scale, $v_a \sim v_w$ the velocity scale (Mandre *et al.* 2009), and $h_a \sim 1 \mu\text{m}$ (van der Veen *et al.* 2012).

For the relaxation stage, the low Reynolds number in the viscous thin film $Re_f = \rho_f L_f v_f \eta_f^{-1} \sim 10^{-2}$ indicates applicability of Stokes flow theory, using $L_f \sim h_f$ and $v_f \sim \gamma_f \eta_f^{-1}$ (Salez *et al.* 2012). Here, $\eta^* \sim 10^2$ and $h^* \sim 10^{-2}$ suggest small amplitude thin film perturbations due to small film thickness in comparison with the lateral length scales ($h_f \ll R_w$). Drop size and film thickness are much below the capillary length, so that gravity effects can be neglected.

3. Deformation of viscous thin films

3.1. Typical bouncing experiment

Before turning to a detailed quantitative analysis, we first describe the oil film deformations observed in a typical experiment. The synchronized recordings of the drop bouncing using a high speed camera and the oil surface deformation measured using DHM are shown in the respective top and bottom rows in figure 3.

When the falling water drop is still far from the oil-air interface, the droplet takes on a spherical shape while there is no deformation in the oil surface (cf. figure 3a & 3e). As the bottom of the falling water drop approaches the oil-air interface, the air pressure builds up in the narrow air gap, deforming both the water-air and oil-air interface. The lubrication air pressures in narrow air gaps can become sufficiently large to decelerate the falling drop, bringing it to rest (or in apparent contact with the oil-air interface) and

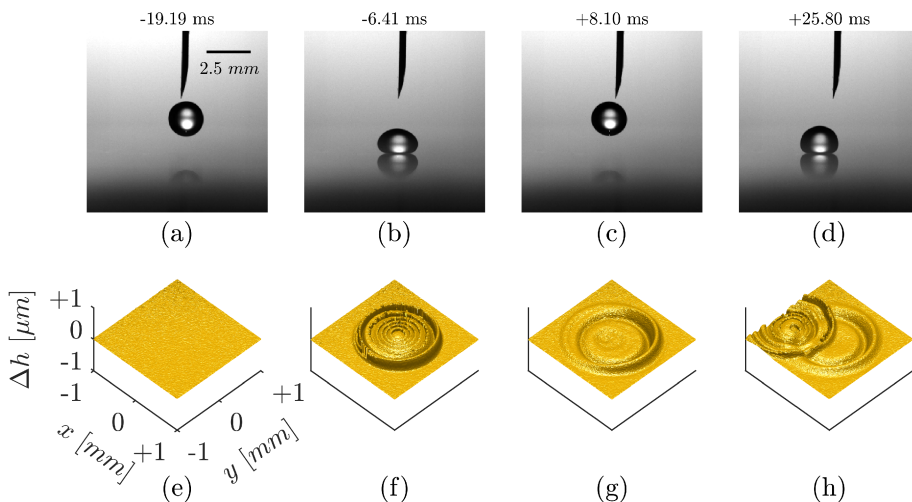


Figure 3: Snapshots of a water drop bouncing on an oil film. Drop bouncing behaviour shown in the first row (a) – (d). Evolution of the oil-air deformation shown in the second row (e) – (h). Location of impact center is $[x, y] = [0, 0]$ mm. The control parameters are $We = 0.38$, $h^* = 0.01$, and $\eta^* = 98$.

cause a reversal in droplet’s momentum, leading to a contact-less drop bouncing. The maximum drop spreading and the corresponding oil-air deformation obtained during the apparent contact is shown in figure 3b & 3f. It should be noted that during this phase the holography measurement cannot be trusted quantitatively. This is due to the fact that when the drop is too close to the oil-air interface (small air gaps, $h_a \lesssim 100 \mu\text{m}$), additional light reflections from the water-air interface interfere with the measurements of the oil-air interface (cf. Appendix A). In particular, the concentric ring structure seen in figure 3f is such an artifact. However, as soon as the drop has bounced back and is sufficiently far away from the oil-air interface (large air gaps, $h_a \gtrsim 100 \mu\text{m}$), light reflections from the water-air interface are no longer present, and the measurements of the oil-air interface are quantitatively accurate. A snapshot of the drop bounced off far away from the oil-air interface and the corresponding oil-air deformation are shown in figure 3c & 3g. Subsequent to this, the oil film relaxes towards a flat state under the influence of surface tension, until it is again perturbed by a second impact of the drop (cf. figure 3d & 3h).

Figure 4a presents a typical surface topography of the oil-air interface after the bounce. The corresponding azimuthally averaged deformation profile is shown in figure 4b, where in this case the average is performed over the full annulus within $0 \leq \theta < 2\pi$. In the remainder, we choose $t = 0$ as the earliest time when clean DHM measurements are obtained after the drop bounce off process (cf. Appendix A). Figure 4a shows that deformations are highly localised, within a narrow annulus $r_{an} \approx 0.6 - 0.8$ mm. Given that flow inside the viscous film requires pressure gradients, such localised deformations suggest that spatial variations of air pressure during the bounce are highly localised during impact (cf. figure 1b). The appearance of such an annulus is reminiscent of dimple formation underneath an impacting drop: An annular local minimum of the air gap is

seen for drops impacting a dry substrate (Mandre *et al.* 2009; Hicks & Purvis 2010; Kolinski *et al.* 2012; van der Veen *et al.* 2012; Bouwhuis *et al.* 2012), drops impacting a thin liquid film (Hicks & Purvis 2011) and drops impacting a liquid pool (Hendrix *et al.* 2016). We therefore hypothesize that the radial location of the deformation correlates to the minimum of the air gap during drop impact.

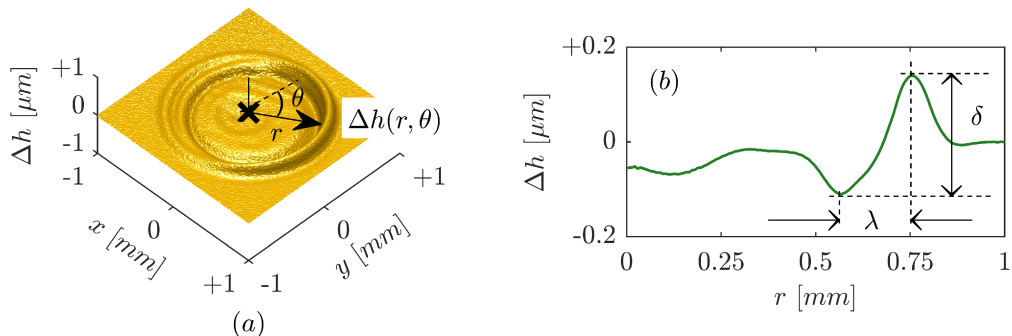


Figure 4: (a) Surface topography of the oil-air interface at $t = 0$. (b) Azimuthally averaged deformation profile over the full annulus at $t = 0$. We define the wave characteristics δ (amplitude) and λ (wavelength). The control parameters are $We = 0.38$, $h^* = 0.01$, and $\eta^* = 98$.

In the present experiments, the oil surface deformations at $t = 0$ are not perfectly axisymmetric (cf. figure 4a). This small asymmetry is attributed to a small horizontal impact speeds, which is difficult to eliminate experimentally. This small horizontal speed affects the air layer thickness during the bounce process (Lo *et al.* 2017), leaving asymmetric imprint on the oil layer. In the remainder we will average the profiles only over one quadrant. Finally, we remark that figure 4b defines two quantities that will be used below to characterise the wave: the amplitude δ and the wavelength λ , respectively defined as the vertical and horizontal distance from the minimum to the maximum of the film.

3.2. Influence of film properties and drop impact velocity

We now study the influence of the film thickness and the film viscosity on the surface deformations left behind after impact. Figures 5a - 5d show the surface topographies at $t = 0$ in one quadrant. Figures 5e and 5f show the corresponding azimuthally averaged deformation profiles at $t = 0$, averaged over the quadrant. Clearly, a decrease in deformation amplitude δ is seen with a decrease in initial film thickness (cf. rows in figure 5a - 5d and figure 5e). On the other hand, a decrease in deformation amplitude is seen with an increase in film viscosity (cf. columns in figure 5a - 5d and figure 5f). To further quantify this, we plot the initial amplitude $\delta_0 = \delta(t = 0)$ as functions of film thickness and film viscosity in figure 6a and 6b. From these plots we empirically deduce that the scaling of the initial amplitude is consistent with $\delta_0 \sim h_f^2 \eta_f^{-1}$, though the data only cover less than one decade in h_f and η_f . The $\delta_0 \sim h_f^2 \eta_f^{-1}$ scaling is not immediately obvious, since the ‘‘mobility’’ of a thin layer flow is known to scale as $h_f^3 \eta_f^{-1}$ (Oron *et al.* 1997).

Next, we study the influence of the impact velocity on the surface deformations left behind after impact. Figures 7a & 7b show the surface topographies at $t = 0$ for

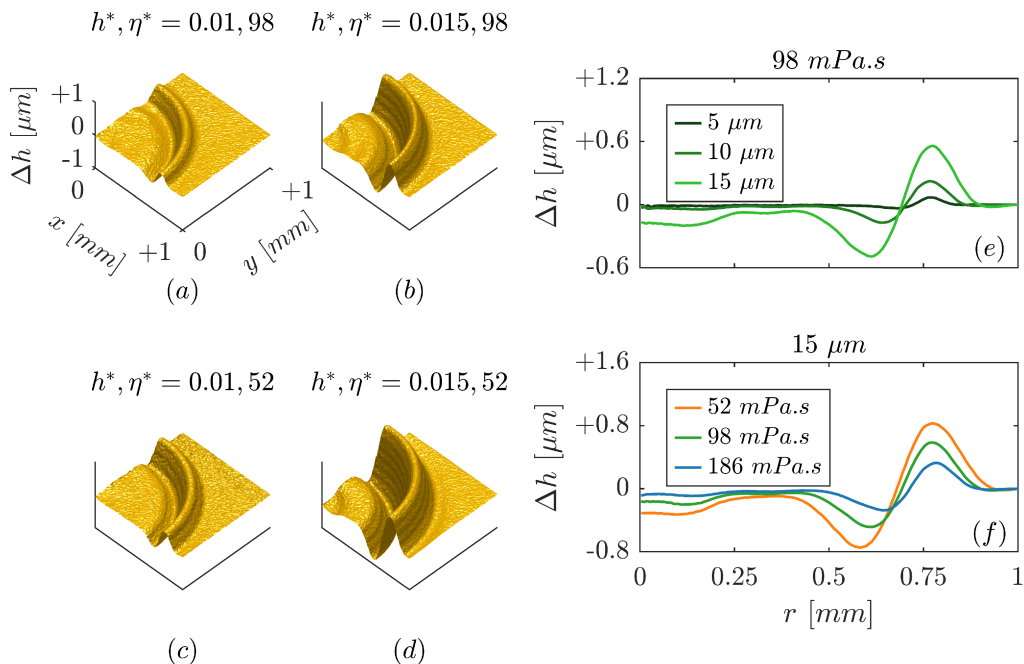


Figure 5: (a) - (d) Surface topographies of the oil-air interface at $t = 0$ in a quadrant $0 \leq \theta < \pi/2$. (e) & (f) Azimuthally averaged deformation profile over the quadrant at $t = 0$. The control parameters are $We = 0.38$, $h^* = 0.005 - 0.015$, and $\eta^* = 52 - 186$.

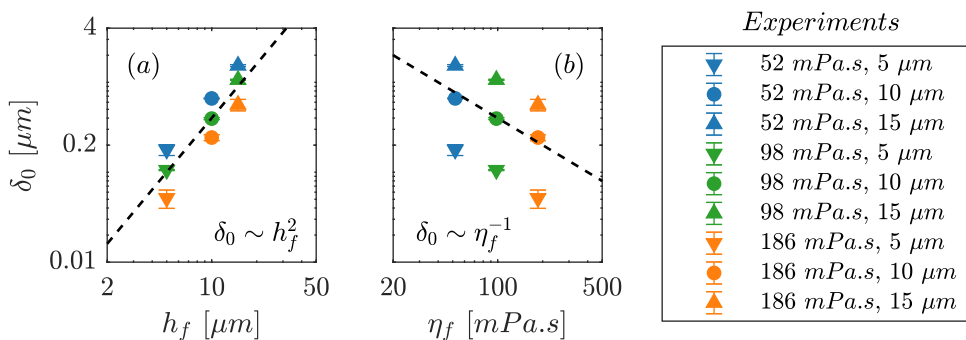


Figure 6: Scaling of the initial amplitude with film thickness and film viscosity. The mean and errorbar values of δ_0 are based on 3 experimental repeats. The control parameters are $We = 0.38$, $h^* = 0.005 - 0.015$, and $\eta^* = 52 - 186$.

two different impact velocities. Figure 7c shows the corresponding azimuthally averaged deformation profiles. For the higher impact velocity (cf. figure 7b), two distinct peaks in deformation are observed – we emphasise that the profile corresponds to a single impact. This is in contrast with the single peak that appears at lower velocity (cf. figure 7a). Moreover, the deformations are more radially spread out for the higher impact speed as compared to lower impact speed. The transition from one peak to two peaks and the increased radial spread is reminiscent of the transition from single dimple to double dimple formation in a falling drop, as previously observed on dry surfaces (de Ruiter *et al.*

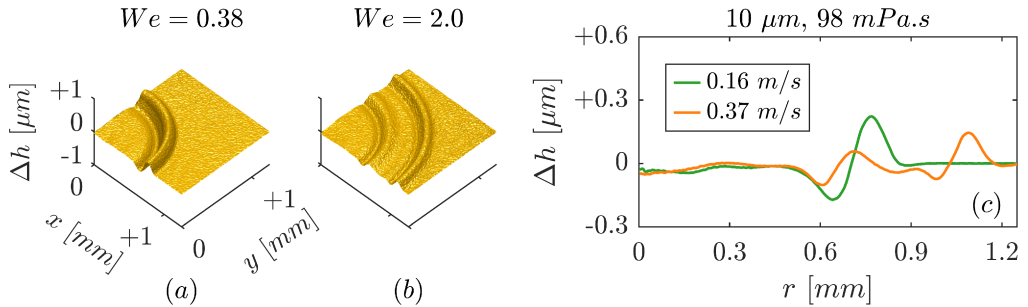


Figure 7: (a) - (b) Surface topographies of the oil-air interface at $t = 0$ in a quadrant $0 \leq \theta < \pi/2$. (c) Azimuthally averaged deformation profile over the quadrant at $t = 0$. The control parameters are $We = 0.38 - 2.0$, $h^* = 0.01$, and $\eta^* = 98$.

2012, 2015). This again suggests that the deformation directly reflects the structure of the dimple below the impacting drop.

4. Relaxation of viscous thin film deformations

4.1. Spacetime plot of typical relaxation process

We now reveal the relaxation of viscous thin film deformations after the impact process. When the drop is far away from the film surface after the bounce, the air pressure is again homogeneous and no longer provides any forcing to the film. By consequence film deformations gradually decay via an intricate relaxation process, under the influence of surface tension. Figure 8 provides a spacetime plot of a typical relaxation process, over two decades in time ($t \sim 0.01 - 1 \text{ s}$). The figure corresponds to an azimuthal average of surface deformation within a quarter annulus ($0 \leq \theta < \pi/2$). The lines indicate the loci of deformation maxima (orange), minima (blue) and zero-crossing (green). These lines highlight that the deformation involves a decay in amplitude as well as a broadening of the lateral width of the deformation profile. Note that during this process, the position of the zero crossing (green line) remains approximately constant.

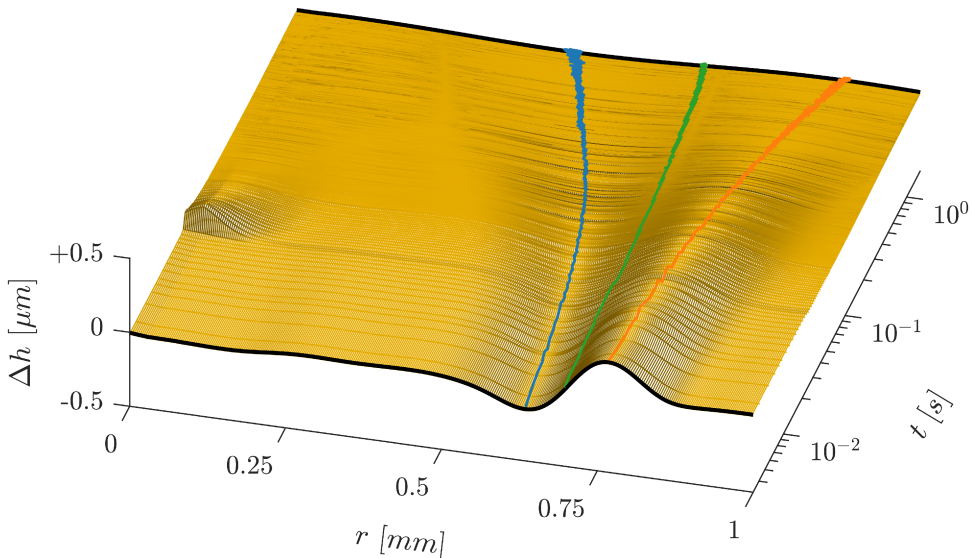


Figure 8: Spacetime plot of the relaxation process. Initial and final deformations are plotted as black lines. Loci of deformation maxima, minima and zero crossings are plotted as orange, blue, and green lines, respectively. A secondary deformation occurs at $t \approx 25$ ms near the impact center due to the next impact process. The control parameters are $We = 0.38$, $h^* = 0.01$, and $\eta^* = 98$.

4.2. Numerical simulation

We perform numerical simulations in order to study the relaxation process of the viscous thin films. The relaxation process is modelled using lubrication theory (Reynolds 1886; Oron *et al.* 1997). Lubrication theory relies on the following conditions, which are indeed satisfied in the experiment, namely, (i) viscous forces in the film dominate over inertial forces ($Re_f \sim 10^{-2} \ll 1$) and (ii) deformation amplitudes in vertical direction are much lower than the characteristic lateral length scale ($\delta/\lambda \sim 10^{-2} \ll 1$). As boundary conditions we consider the free surface to be in contact with a homogenous gas pressure, as is the case after rebound, while there is a no-slip boundary condition at the substrate. The corresponding lubrication (thin film) equation reads (Oron *et al.* 1997):

$$\partial_t h + \frac{\gamma_f}{3\eta_f} \vec{\nabla} \cdot \left\{ h^3 \vec{\nabla} \left(\vec{\nabla}^2 h \right) \right\} = 0, \quad (4.1)$$

where $h(x, y, t)$ is the vertical distance between the solid substrate and the free surface and $\vec{\nabla}$ is the two-dimensional gradient operator in the x-y plane. We perform a nondimensionalisation of (4.1) by $h = h_f H$, $r = h_f R$ and $t = (3\eta_f h_f \gamma_f^{-1}) T$, where h_f is the initial film thickness. In the following, we will study the relaxation process in an axisymmetric geometry, i.e. $H = H(R, T)$ such that (4.1) becomes :

$$\partial_T H + \frac{1}{R} \partial_R \left[R H^3 \left(\partial_R^3 H + \frac{1}{R} \partial_R^2 H - \frac{1}{R^2} \partial_R H \right) \right] = 0. \quad (4.2)$$

Importantly, (4.2) is devoid of any free parameters. To compare to experiment, we perform numerical simulation of film relaxation using a finite element method and a second-order implicit Runge-Kutta time-stepping scheme (implemented using the framework DUNE-PDELAB by Bastian *et al.* (2008*b,a*, 2010)). The deformation profile at $t = 0$

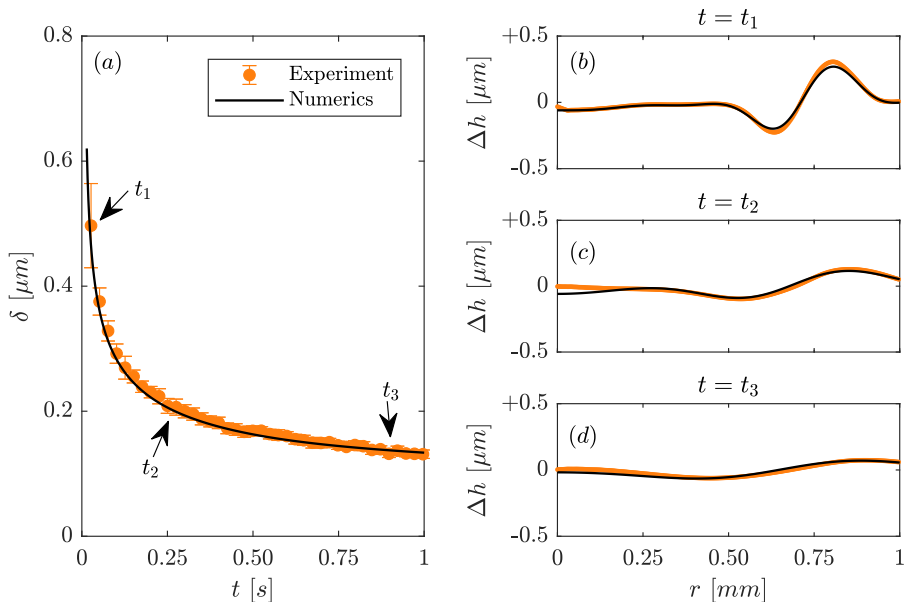


Figure 9: (a) Comparison of amplitude decay between experiment and numerics. The mean and errorbar values of δ are obtained by binning every 25 datapoints. (b – d) Comparison of film deformation between experiment and numerics at $t_1 = 14 \text{ ms}$, $t_2 = 252 \text{ ms}$ and $t_3 = 966 \text{ ms}$. The control parameters are $We = 0.38$, $h^* = 0.01$, and $\eta^* = 52$.

is taken from the experiment and used as an initial condition, and subsequently the film profile is evolved via numerical integration of (4.2). A direct comparison between the experiments and the lubrication theory is given in figure 9 without any adjustable parameters. Figure 9a shows the amplitude δ (defined as the difference between maximum and minimum of Δh) as a function of time, while figure 9b shows the deformation profiles $\Delta h = h(r, t) - h_f$ at different times. The comparison exhibits very good agreement between the experiment and the numerical calculations, demonstrating the success of the lubrication approximation to describe the experimentally observed relaxation.

4.3. Theoretical analysis

Now we turn to a detailed theoretical analysis of the relaxation process, from which we will establish the general scaling laws of the relaxation. To do this, we reduce the lubrication equation to a one-dimensional geometry $h = h(x, t)$. Here, we use $X = x/h_f$ analogous to $R = r/h_f$. The rationale behind choosing a 1D lubrication equation is that the initial deformations are far from the impact center (cf. figure 5). This is further quantified in figure 8, where the “width” of the profile is initially an order of magnitude smaller than the location of the zero crossing. Therefore the axisymmetric relaxation and a one-dimensional relaxation will yield very similar results – at least until the deformations approach the impact center and the azimuthal contributions become important.

The one-dimensional lubrication equation reads

$$\partial_T H + \partial_X (H^3 \partial_X^3 H) = 0. \quad (4.3)$$

To further simplify the analysis, we use the fact seen in the experiments that the deformation amplitudes are small in comparison to initial film thicknesses ($\delta/h_f \sim$

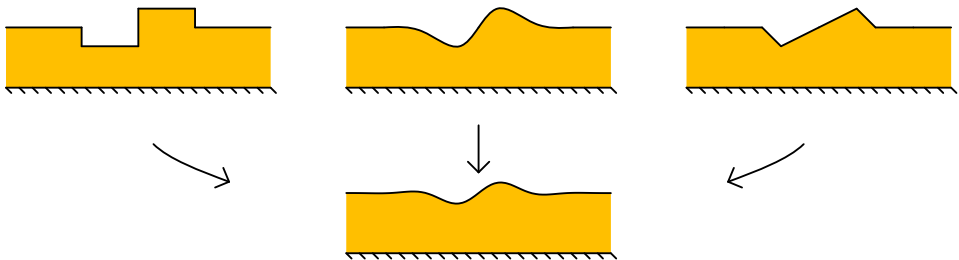


Figure 10: Schematic illustrating the approach to the attractor function $\phi_1(U)$. The initial deformations are zero volume perturbations having moments $\mathcal{M}_0 = 0$ and $\mathcal{M}_1 \neq 0$. Adapted from [Benzaquen *et al.* \(2014, 2015\)](#).

$0.05 \ll 1$). This allows us to linearise (4.3) employing the variable transformation $Z = h/h_f - 1 = H - 1$ where $|Z| \ll 1$. The linearised 1D lubrication equation then reads

$$\partial_T Z + \partial_X^4 Z = 0. \quad (4.4)$$

The relaxation of localised thin film perturbations described by (4.4) was analysed in great detail by [Salez *et al.* \(2012\)](#); [McGraw *et al.* \(2012\)](#); [Bäumchen *et al.* \(2013\)](#); [Backholm *et al.* \(2014\)](#); [Benzaquen *et al.* \(2013, 2014, 2015\)](#); [Bertin *et al.* \(2020\)](#). They obtained the long time asymptotic solution in terms of a moment expansion,

$$Z(X, T) = \frac{\mathcal{M}_0 \phi_0(U)}{T^{1/4}} + \frac{\mathcal{M}_1 \phi_1(U)}{T^{2/4}} + \frac{1}{2} \frac{\mathcal{M}_2 \phi_2(U)}{T^{3/4}} + \dots, \quad (4.5)$$

which involves the similarity variable

$$U = XT^{-1/4}, \quad (4.6)$$

and similarity functions $\phi_n(U)$ that can be determined analytically ([Benzaquen *et al.* 2013, 2014, 2015](#)). The amplitudes \mathcal{M}_n appearing in (4.5) can be determined from the initial condition $Z_0(X) = Z(X, 0)$, by computing the moments

$$\mathcal{M}_n = \int \xi^n Z_0(\xi) d\xi, \quad n = 0, 1, 2, \dots \quad (4.7)$$

It is clear from (4.5) and the similarity variable (4.6) that the width λ of the profile follows a universal scaling of the form $\lambda \sim T^{1/4}$. The decay of the amplitude δ is more subtle, since each term in (4.5) decays differently, as $\delta \sim T^{-(n+1)/4}$ for the n th moment. At late times, the solution $Z(X, T)$ thus converges towards the lowest order term with a non-zero moment. Generically, for $\mathcal{M}_0 \neq 0$, the amplitude will therefore decay as $T^{-1/4}$. In our case, however, the perturbation originates from an initially flat film, and by incompressibility of the layer, the perturbation is thus expected to have a vanishing volume, i.e. $\mathcal{M}_0 = 0$. In the present context, the lowest order moment is therefore expected to be $\mathcal{M}_1 \neq 0$. In this scenario, the scaling law will be $\delta \sim T^{-1/2}$, while the solution $Z(X, T)$ should converge to $\phi_1(U)$ for a zero volume perturbation. A schematic depiction of the approach to the $\phi_1(U)$ attractor is shown in figure 10.

To verify this scenario, we turn to an exemplary initial deformation profile of a 98 *mPa.s*, 10 μm film, and probe the subsequent relaxation. For the specific example, the two lowest order moments are determined as $\mathcal{M}_0 \approx 6.4 \times 10^{-2}$ and $\mathcal{M}_1 \approx 3.2$. The very small value of \mathcal{M}_0 is of the order of the experimental resolution (i.e. it corresponds

to a typical $\Delta h \sim \pm 10$ nm), so that indeed the perturbation has a negligible volume. Figure 11 reveals that the relaxation is indeed governed by \mathcal{M}_1 , and approaches the $\phi_1(U)$ self-similar attractor function (cf. Appendix B). The figure reports the scaled deformation profiles centred around the zero crossing location $X_0 \approx 65$. The first row shows the deformation profile scaled with the initial film thickness. The second row shows the rescaled deformation profiles (vertical scale $\sim T^{-1/2}$ and horizontal scale $\sim T^{1/4}$). During late times, the rescaled deformation profiles clearly approach the attractor function $\phi_1(U)$, which is superimposed on the data. This excellent match confirms that, within the experimental times, the axisymmetric effects have not yet started to contribute and the theoretical analysis of the film relaxation process over a 1D geometry is sufficient to understand the relaxation process in the experiments.

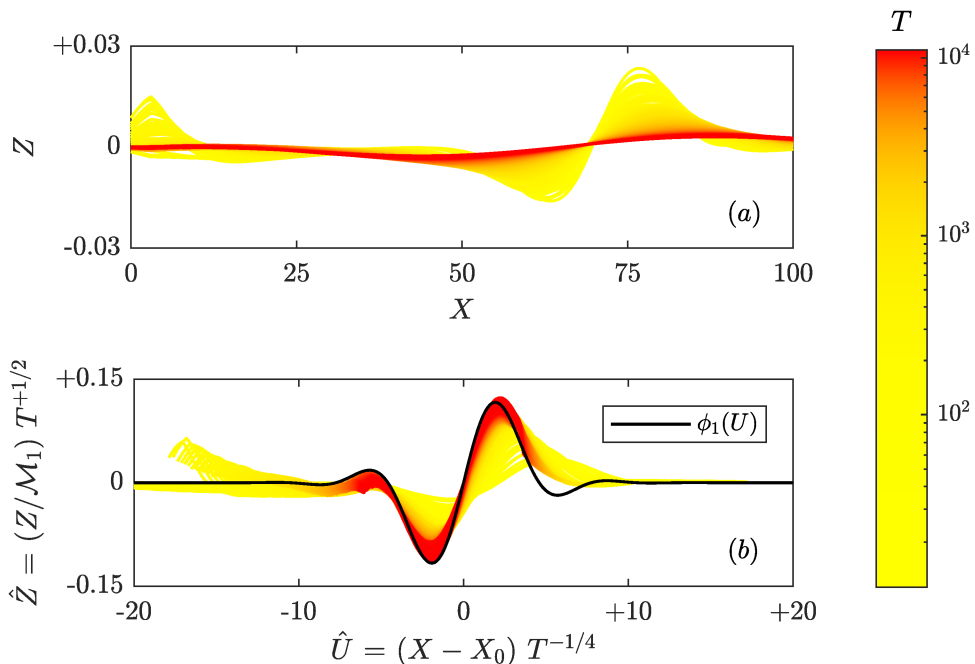


Figure 11: (a) Time evolution of the normalized deformation profiles Z vs X . A secondary deformation occurs at $T \approx 160$ near the impact center due to the next impact process (cf. figure 8). (b) Time evolution of the scaled normalized deformation profiles \hat{Z} vs \hat{U} . The self-similar attractor function $\phi_1(U)$ is plotted as a black line. Here, $X_0 \approx 65$ and $\mathcal{M}_1 \approx 3.2$. The scaled and rescaled deformations are color coded with time; yellow to red as time increases. The control parameters are $We = 0.38$, $h^* = 0.01$, and $\eta^* = 98$.

4.4. Width broadening and amplitude decay

Finally, we will compare theoretical asymptotic scaling laws for the width broadening and amplitude decay with a large number of experiments, all attained for a drop impact velocity of $v_w \approx 0.16$ m/s. We predict the scaling law for the width λ quantitatively, based on the approach to the attractor function ϕ_1 . We formally define the half-width of the similarity function as $U_1^* = \arg \max |\phi_1(U)| \approx 1.924$, which is half the absolute distance between global maxima and global minima (cf. figure 11). From (4.6), we then find

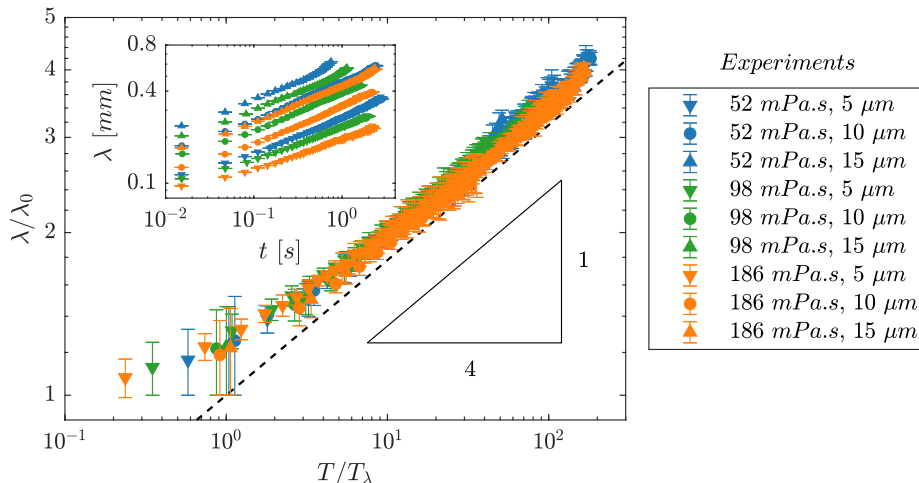


Figure 12: Double logarithmic plot of λ/λ_0 vs T/T_λ . The mean and errorbar values of λ/λ_0 are obtained by binning every 25 datapoints. The black dashed line represents $\lambda/\lambda_0 = (T/T_\lambda)^{1/4}$. The control parameters are $We = 0.38$, $h^* = 0.005 - 0.015$, and $\eta^* = 52 - 186$.

$$\frac{\lambda}{2h_f} \simeq U_1^* T^{1/4}, \quad (4.8)$$

expressing the dimensionless (half) width of the decaying profiles. A practical problem arises when comparing to experiment: at $t = 0$, the width takes on a finite value λ_0 , so that it is initially incompatible with the form (4.8). To resolve this, we follow [Benzaquen *et al.* \(2015\)](#) and define for each experiment a convergence time T_λ through $\lambda_0/2h_f = U_1^* T_\lambda^{1/4}$. The physical meaning of T_λ is that it provides a time at which the experiment should approach the asymptotic power law (4.8). Using this definition of T_λ , the scaling (4.8) then gives

$$\lambda/\lambda_0 \simeq (T/T_\lambda)^{1/4}, \quad (4.9)$$

which can be compared to experiments without adjustable parameters.

In figure 12, we show the temporal dependence λ/λ_0 vs T/T_λ for different initial film thicknesses and viscosities. The black dashed line in the figure represents Eqn (4.9). Clearly, all experimental data points seem to collapse onto a single master curve which is independent of the film properties used. Moreover, the master curve has a very good agreement with Eqn (4.9). We remark that such a scaling $\lambda \sim t^{1/4}$ is also seen in previous studies with viscous thin film configurations ([Salez *et al.* 2012](#); [McGraw *et al.* 2012](#); [Benzaquen *et al.* 2015](#); [Hack *et al.* 2018](#)).

A similar analysis is performed for the amplitude decay. We will once again make use of the self-similar attractor $\phi_1(U)$ for the relaxation process. Similar to the procedure outlined for the study of the width broadening, we employ Eqn (4.5) which implies,

$$\frac{\delta}{2h_f} \simeq \mathcal{M}_1 \frac{|\phi_1(U^*)|}{T^{1/2}}, \quad (4.10)$$

where $|\phi_1(U^*)| \approx 0.1164$ is the maximum of the similarity function (cf. figure 11). To

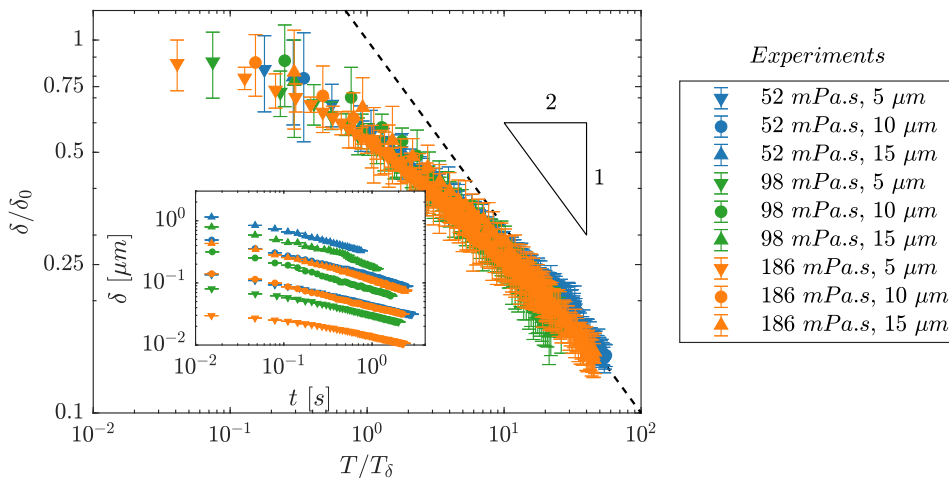


Figure 13: Double logarithmic plot of δ/δ_0 vs T/T_δ . The mean and errorbar values of δ/δ_0 are obtained by binning every 25 datapoints. The black dashed line represents $\delta/\delta_0 = (T/T_\delta)^{-1/2}$. The control parameters are $We = 0.38$, $h^* = 0.005 - 0.015$, and $\eta^* = 52 - 186$.

avoid the experimental issue that δ_0 , the amplitude at $t = 0$, is finite, we once again determine for each experiment a convergence time T_δ , using $\delta_0/(2h_f) = \mathcal{M}_1|\phi_1(U^*)|/T_\delta^{1/2}$. Note that both δ_0 and \mathcal{M}_1 will be different for each specific experiment, but both parameters can be determined independently. This finally gives that (4.10) can be written as

$$\delta/\delta_0 \simeq (T/T_\delta)^{-1/2}. \quad (4.11)$$

The result for δ/δ_0 vs T/T_δ for different film thicknesses and different viscosities are shown in figure 13. The black dashed line in the figure represents Eqn (4.11). While from the individual measurements it is difficult to infer the $1/2$ scaling law, it is clear that the dashed line captures the correct asymptotics.

5. Conclusions and Outlook

In this work, we have performed experiments of a water drop impacting a viscous thin oil film in an ambient air environment. The considered drop impact velocities are restricted to moderately low values $We \sim 1$ at which drops bounce on thin films, due to the air cushioning effect. Digital Holographic Microscopy was employed to measure the deformations of the free oil film surface that arise due to the bouncing of drops with an unprecedented precision, allowing for the one-to-one comparison with lubrication theory.

We first investigated the deformations of the thin film immediately after rebound ($t = 0$) while varying the oil film thickness h_f , oil viscosity η_f , and the impact velocity v_w of the drop. We found that the deformation amplitude after the bounce δ_0 varies quadratically with oil film thickness $\delta_0 \sim h_f^2$ and inversely with the oil viscosity $\delta_0 \sim \eta_f^{-1}$. When increasing the impact speed from $v_w \approx 0.16$ m/s to $v_w \approx 0.37$ m/s, the deformations in the thin film change even qualitatively: While at lower speeds, a single annular wavy

deformation is found, at higher speeds the radial profiles exhibit two of such depressions and peak at two different radii.

In the second part of the manuscript, we have detailed the relaxation process of the viscous thin films when the drop is far away from the free oil surface after the bounce. Numerical calculations based on lubrication theory using the experimental deformations at $t = 0$ as initial condition show an excellent match with the experimentally measured evolution of the deformation profiles. Furthermore, we have successfully employed a theoretical analysis developed by [Benzaquen *et al.* \(2013, 2014, 2015\)](#) to obtain analytical results describing the relaxation process: Taking advantage of the fact that the deformations approach a universal self-similar attractor, at late times of the relaxation process the decay of the amplitude and the growth of the width of the deformations can be described without any free parameters. This allows us to collapse the corresponding experimental curves for all different thin film properties investigated.

Measuring the deformations of the falling drop and the viscous thin film simultaneously has proven challenging in the previous literature ([Lo *et al.* 2017](#)) and in the present experiments. However it is worthwhile to investigate the dynamics of the coupled system as it would allow valuable insight in the formation mechanism of the deformations. To resolve both the drop-air and the oil-air interface, we therefore plan to combine the color interferometry technique by [van der Veen *et al.* \(2012\)](#), which can be used to extract the narrow air profiles, with the DHM technique described in the present manuscript. In order to understand the film deformations theoretically and quantify possible influences on the macroscopic drop dynamics, the macroscopic impact dynamics have to be coupled to a two layer lubrication model for the air layer and the lubrication layer. Very recently, such a lubrication approach has been pursued by [Duchemin & Josserand \(2020\)](#) for the coalescence of a drop with an underlying thin film and when combining color interferometry with DHM it can be tested against controlled experiments.

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Appendix A. Measuring thin film deformations using DHM

Figure 14a shows a schematic diagram where the drop has just moved out of the DHM measuring window after the bounce. The height of the measuring window is roughly $200 \mu\text{m}$. Here, we define $t = 0$ as the first instance when the DHM measuring window is devoid of the water-air interface allowing for a clean measurement of the oil-air deformation. The object beam (consisting of reflection wavefronts from the measuring window) and the reference beam in the DHM interfere to form the holographic pattern recorded by the DHM camera.

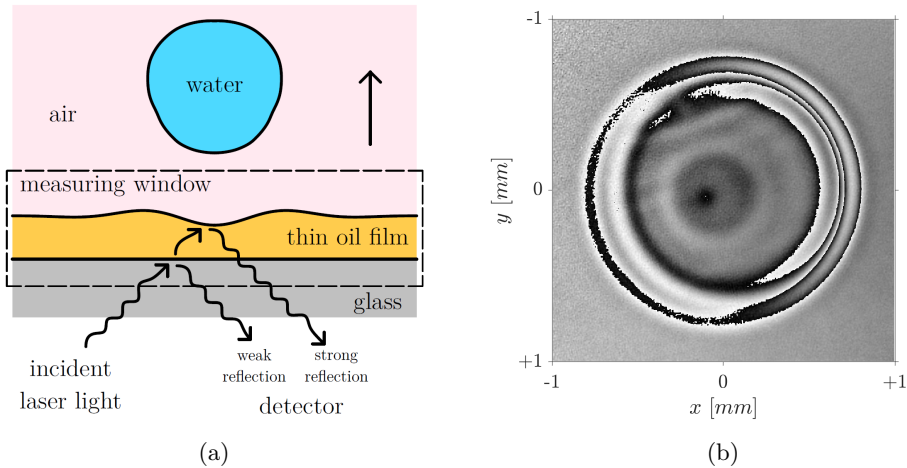


Figure 14: (a) Schematic diagram at $t = 0$ when the drop has just left the DHM measuring window after the bounce. (b) Phase image of the oil-air deformation at $t = 0$. The control parameters are $We = 0.38$, $h^* = 0.01$, and $\eta^* = 98$.

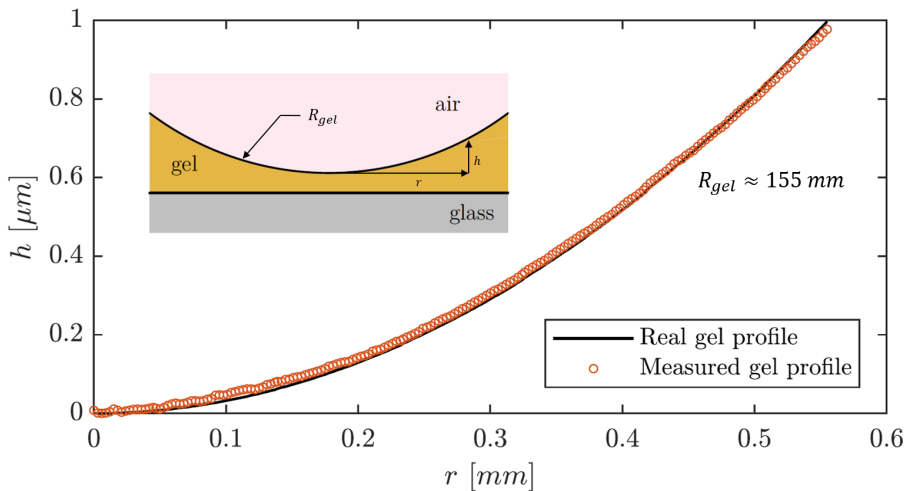


Figure 15: Comparison of thin PDMS gel profiles of a known curvature.

The recorded holograms are converted into their constituent intensity and phase information, wherein the phase information is used to reconstruct the height information. Figure 14b shows an exemplary phase image at time $t = 0$. The reconstructed height profile of this phase image is shown in figure 4a.

To check whether the weak reflection of the glass-oil interface affects the measurements of the oil-air deformation in the bounce experiments, we perform a simple calibration experiment. The calibration experiment is performed to measure the (known) PDMS gel-air profile through glass. Figure 15 shows the comparison of the real and the measured PDMS gel-air deformation. We confirm that the flat glass-PDMS gel interface (and

similarly the flat glass-oil film interface) does not affect the measurements of the thin film deformations. A vertical resolution of around 20 nm is obtained from the calibration plot. It is important to note that the vertical resolution can be well below 20 nm. This is only a conservative estimate since the real PDMS gel profile can suffer from small tilt issues and from surface roughness during its fabrication which are not accounted for here.

Appendix B. Similarity function $\phi_1(U)$

In principle the similarity functions $\phi_n(U)$ are completely covered in the works of Benzaquen *et al.* (2013, 2014, 2015), but here we provide the expression for the similarity function $\phi_1(U)$ as a reference. The expression for $\phi_1(U)$ is given in Eqn (B 1) reads

$$\begin{aligned} \phi_1(U) = & + \frac{U}{4\pi} \Gamma\left(\frac{3}{4}\right) {}_0H_2 \left[; \left\{ \frac{5}{4}, \frac{3}{2} \right\}; \frac{U^4}{256} \right] \\ & - \frac{U^3}{24\pi} \Gamma\left(\frac{5}{4}\right) {}_0H_2 \left[; \left\{ \frac{3}{2}, \frac{7}{4} \right\}; \frac{U^4}{256} \right] \\ & + \frac{U^5}{960\pi} \Gamma\left(\frac{3}{4}\right) {}_0H_2 \left[; \left\{ \frac{9}{4}, \frac{5}{2} \right\}; \frac{U^4}{256} \right]. \end{aligned} \quad (\text{B } 1)$$

Here Γ and ${}_0H_2$ are the gamma function and the (0,2)-hypergeometric function respectively. The (0,2)-hypergeometric function is defined in Eqn (B 2) which reads

$${}_0H_2 [; \{a, b\}; w] = \sum_{n=0}^{\infty} \frac{1}{(a)_n (b)_n} \frac{w^n}{n!}. \quad (\text{B } 2)$$

Here $(.)_n$ is the Pochhammer notation for the rising factorial.

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