

# Soft Wetting: Models based on energy dissipation or on force balance are equivalent

Stefan Karpitschka<sup>1</sup>, Siddhartha Das<sup>2</sup>, Mathijs van Gorcum<sup>3</sup>, Hugo Perrin<sup>4</sup>, Bruno Andreotti<sup>4</sup>, and Jacco H. Snoeijer<sup>3</sup>

<sup>1</sup>*Department Dynamics of Complex Fluids, Max Planck Institute for Dynamics and Self-Organization (MPIDS), Am Fassberg 17, 37077 Goettingen, Germany.*

<sup>2</sup>*Department of Mechanical Engineering, University of Maryland, College Park, Maryland 20742, USA.*

<sup>3</sup>*Physics of Fluids Group and J. M. Burgers Centre for Fluid Dynamics, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands.*

<sup>4</sup>*Laboratoire de Physique Statistique (LPS), UMR 8550 CNRS, ENS, Univ. Paris Diderot, Sorbonne Université, 24 rue Lhomond, 75005, Paris, France.*

In Newtonian mechanics, an overdamped system at steady-state is governed by a local balance of mechanical stress, but also obeys a global balance between injected and dissipated energy. In the classical literature of purely viscous drop spreading, apparent differences in “dissipation” and “force” approaches have led to unnecessary debates, which ultimately could be traced back to different levels of mathematical approximation [1]. In the context of wetting on a soft solid, Zhao *et al.* [2] interpret their experiments by a model based on viscoelastic dissipation inside the substrate. It is claimed that this global dissipation model is fundamentally different from the local mechanical model presented by Karpitschka *et al.* [3]. The purpose of this Letter is to demonstrate that: (i) The models by [2] and [3] are in fact strictly equivalent, (ii) The apparent difference can be traced back to an inconsistent approximation made in [2].

Following the analysis in [2], there is a step where the dissipation per unit volume is integrated over depth (equation 44 to 45 of the Supplement). The analysis provides no information on the explicit depth-dependence; the integral is estimated to scale with the wavenumber as  $1/k$ , arguing that this is the extent by which the deformation penetrates into the layer. Such an approximation is inconsistent, however, since the finite thickness  $h_0$  induces a screening of the modes of wavenumber  $k \lesssim 1/h_0$ . This is a key point, since this estimation underlies the scaling laws presented in the main text.

To resolve this issue explicitly, we propose to perform the depth-integral at the very start of the analysis, and compute the dissipation  $P$  as

$$P = \int d^2x \sigma_{ij} \frac{\partial \dot{u}_i}{\partial x_j} = \oint ds \sigma_{ij} n_j \dot{u}_i = \int_{-\infty}^{\infty} dx \sigma(x, t) \dot{h}(x, t). \quad (1)$$

In the last step, and for the rest of the analysis, we strictly follow [2] by keeping track only of the normal displacement  $h(x - vt)$  and the normal traction  $\sigma(x - vt)$ . We then proceed with the exact same formula for  $\sigma$  proposed in [2, 3], and obtain an expression in terms of the Fourier transform  $h(k)$ :

$$P = -v \int \frac{dk}{2\pi} \frac{\mu(kv)}{K(k)} (ik) |h(k)|^2 = v(\gamma \sin \theta)^2 \int \frac{dk}{2\pi} \frac{k K(k) G''(kv)}{|K(k) \gamma_s k^2 + \mu(kv)|^2} \quad (2)$$

where  $\mu(\omega) = G'(\omega) + iG''(\omega)$  is the viscoelastic modulus and  $K(k)$  is the spatial Green’s function that accounts for the finite layer thickness. In the second step we used the explicit form of  $h(k)$ . This result differs from the estimation given in equation 45 in the SI of [2]. However, balancing this explicitly integrated formula for the dissipation with the power injected by capillary forces, one exactly recovers equations 22-23 in [3], even up to the prefactors.

As expected, the dissipation route proposed in [2] leads, once derived consistently, to the same prediction as the mechanical approach [3] under the same assumptions: small solid surface deformations in order to use the Green’s function formalism, and constant solid surface tension, decoupled from strain. For fully quantitative comparison with experiments, future work should go beyond these restrictions.

---

[1] D. Bonn, J. Eggers, J. Indekeu, J. Meunier, and E. Rolley. Wetting and spreading. *Rev. Mod. Phys.*, 81:739, 2009. doi: 10.1103/RevModPhys.81.739.

[2] S Zhao, J Dervaux, T Narita, F Lequeux, L Limat, and M Roché. Geometrical control of dissipation during the spreading of liquids on soft solids. *Proc. Natl. Acad. Sci. U.S.A.*, 115(8):1748–1753, 2018. doi: 10.1073/pnas.1712562115.

[3] S. Karpitschka, S. Das, M. van Gorcum, H. Perrin, B. Andreotti, and J.H. Snoeijer. Droplets move over viscoelastic substrates by surfing a ridge. *Nat. Commun.*, 6:7891, 2015. doi: 10.1038/ncomms8891.